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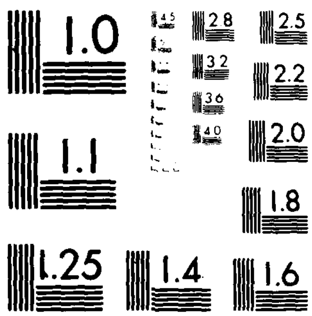

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On Kraichnan's "Direct Interaction Approximation" and Kolmogoroff's Theory in Two-Dimensional Plasma Turbulence

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On Kraichnan's "Direct Interaction Approximation" and Kolmogoroff's Theory in Two-Dimensional Plasma Turbulence

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The nonlinear damping in a strongly turbulent convecting plasma computed by Kraichnan's modified direct interaction approximation and the power spectrum are rederived in a physically transparent form using Kolmogoroff's theory of turbulence.

Recently Sudan and Keskinen[1,2] have developed a two-dimensional theory of strong, low-frequency electrostatic turbulent convection in a collisional, low pressure plasma by means of Kraichnan's[3] modified[4] "direct interaction approximation" (DIA). This theory explains[5] many of the features of the electron density fluctuations in the equatorial electrojet observed by radar scattering techniques.[6] It is the purpose of the present note to show that the principal results of the DIA formalism applied to the electrojet driven electrostatic turbulence can be derived in a much simpler and physically intuitive manner from Kolmogoroff's theory of turbulence.[7]

The equatorial electrojet region can be modeled by a weakly ionized plasma in a strong uniform magnetic field $B\hat{x}$ subjected to both a gradient in density $(\partial n_o/dz)\hat{z}$ and an electric field $\underset{\sim}{E}_o = -(\partial\phi_o/\partial z)\hat{z}$. The electrons and ions suffer collisions mainly with the background neutrals such that $\Omega_e >> \nu_e$ and $\Omega_i \leq \nu_i$, where $\Omega_{e,i}$ and $\nu_{e,i}$ are the respective cyclotron and collisional frequencies. Because of the difference in collisionality of the electrons and ions the equilibrium field $\underset{\sim}{E}_o$ gives rise to a current density $J_o\hat{y}$ resulting from $\underset{\sim}{E}_o \times \underset{\sim}{B}/B^2$ electron drift. Simon[7] and Hoh[8] have shown that this configuration is unstable to electrostatic fluctuations $\phi(z)\ \mathrm{exp}\, i(k_y y-\omega t)$ if $\nabla n_o\cdot\nabla(-e\phi_o) > 0$ which situation is obtained in the electrojet during the daytime.[5,6] This instability is analogous to the Rayleigh-Taylor instability by replacing the gravitational potential with $-e\phi_o$. The nonlinear interaction of the waves creates an almost isotropic turbulence

in the plane perpendicular to the magnetic field, and reaches a steady state for a steady electrojet current density $\underset{\sim}{J}_o = n_e e(E_o/B_o)\hat{y}$.

The electrojet region is well described by the two-fluid equations applied to an isothermal, quasineutral plasma neglecting both electron and ion inertia for the low frequency fluctuations under discussion. The linearized form of these equations are

$$\frac{\partial n^{(1)}}{\partial t} + \underset{\sim}{v}_o \cdot \nabla n^{(1)} + \underset{\sim}{v}_e^{(1)} \cdot \nabla n_o + n_o \nabla \cdot \underset{\sim}{v}_e^{(1)} = 0, \tag{1}$$

$$e(\underset{\sim}{E}^{(1)} + \underset{\sim}{v}_e^{(1)} \times \underset{\sim}{B}_o) + T_e \nabla n^{(1)}/n_o + m_e \nu_e \underset{\sim}{v}_e^{(1)} = 0, \tag{2}$$

$$-Ze(\underset{\sim}{E}^{(1)} + \underset{\sim}{v}_i^{(1)} \times \underset{\sim}{B}_o) + T_i \nabla n_i^{(1)}/n_o + m_i \nu_i \underset{\sim}{v}_i^{(1)} = 0, \tag{3}$$

$$\nabla \cdot \underset{\sim}{J}^{(1)} = \nabla \cdot [n_o e(Z\underset{\sim}{v}_i^{(1)} - \underset{\sim}{v}_e^{(1)}) - n^{(1)} e \underset{\sim}{v}_o] = 0, \tag{4}$$

where $\underset{\sim}{v}_o = E_o/B_o\hat{y}$ is the electrojet velocity, $\underset{\sim}{E}^{(1)} = -\nabla\phi$, $n_e^{(1)} = Zn_i^{(1)} \equiv n^{(1)}$, T_e, T_i are the electron ion temperatures and the rest of the notation is standard. In equilibrium $n_o(z) = N_o(1 + z/L)$, $d\phi_o/dz = (\overline{\phi}_o/L)(1 + z/L)$ and L is the electron density vertical scale height. The density fluctuation $n^{(1)} = \delta n \exp i\{(k_y y + k_z z) - i(\omega_k + i\gamma_k)t\}$, with $\underset{\sim}{k} \cdot \underset{\sim}{B} = 0$, and $k_y L >> 1$, has a frequency and damping/growth rate given by

$$\omega_{k_r} = \underset{\sim}{k} \cdot \underset{\sim}{v}_o/(1 + \psi), \tag{5a}$$

$$\gamma_k = \psi(1 + \psi)^{-1} \left(\frac{\Omega_e}{\nu_e} \frac{k_y^2 v_o}{k^2 L(1+\psi)} - k^2 C_s^2/\nu_i\right), \tag{5b}$$

where $C_s^2 = (T_e + T_i)/m_i$ and $\psi = \nu_e \nu_i / \Omega_e \Omega_i$.

The nonlinear interaction between these linear modes is strong because these waves are nondispersive. The principal result of wave interaction is a nonlinear self-damping obtained from DIA theory[1,2,4] and expressed by

$$\Gamma_{\underset{\sim}{k},\omega} = - \int d^2k' \, d\omega' \, W_{\underset{\sim}{k},\underset{\sim}{k}-\underset{\sim}{k}'} \, W_{\underset{\sim}{k}-\underset{\sim}{k}',\underset{\sim}{k}} \, I_{\underset{\sim}{k}',\omega'} (\omega-\omega'-\omega_{\underset{\sim}{k}-\underset{\sim}{k}'} + \Gamma_{\underset{\sim}{k}-\underset{\sim}{k}',\omega-\omega'})^{-1}, \quad (6)$$

where the W's are matrix coefficients of the particular model and the ensemble average $\langle n^*_{\underset{\sim}{k},\omega} n_{k',\omega'}/n_o^2 \rangle = I_{\underset{\sim}{k},\omega} \, \delta(\underset{\sim}{k}-k')\delta(\omega-\omega')$. This integral equation for $\Gamma_{k,\omega_{\underset{\sim}{k}}}$ can be solved, for known $I_{\underset{\sim}{k},\omega}$, by expanding $\Gamma_{k-k',\omega-\omega'}$ about $\omega'-\omega'_{k_r} = 0$. The result of this calculation[1,2] is given by

$$\Gamma_k \equiv \int_0^{2\pi} \frac{d\theta}{2\pi i} \Gamma_{\underset{\sim}{k},\omega_{\underset{\sim}{k}}} = \alpha \, \beta V_o k^2 I_k^{1/2}, \quad (7)$$

where $\underset{\sim}{k} = (k,\theta)$, $\beta = \nu_i/\Omega_i(1 + \psi)$, α is a numerical constant of order unity and the power spectrum $I_k \equiv \int d\omega I_{\underset{\sim}{k},\omega}$ is assumed to be isotropic.

The basic premise of Kolmogoroff's mixing length theory is that a particular convective eddy or mode lasts only for the time it takes to traverse its own dimension ℓ, i.e., the damping rate is given by

$$\Gamma \sim \delta v/\ell \sim k\delta v, \quad (8)$$

where δv is eddy velocity. We express $\delta \underset{\sim}{v}$ from the linearized equations

in terms of the mode intensity I_k as follows. From the continuity equation,

$$\left(\frac{\partial}{\partial t} + i\underset{\sim}{k}\cdot\underset{\sim}{v}_o\right)\delta n_k = -i(\omega_{\underset{\sim}{k}} - \underset{\sim}{k}\cdot\underset{\sim}{v}_o)\delta n_k = -\delta\underset{\sim}{v}_k\cdot\nabla n_o - n_o\nabla\cdot\delta\underset{\sim}{v}_k. \quad (9)$$

Taking the curl of the momentum balance equation (2) we obtain

$$\nabla \times (\delta\underset{\sim}{v}_k \times \underset{\sim}{B}_o) = -\frac{m}{e}\nu_e \nabla \times \delta\underset{\sim}{v}_k$$

or

$$\underset{\sim}{B}_o \nabla\cdot\delta\underset{\sim}{v}_k = -\frac{m}{e}\nu_e \nabla \times \delta\underset{\sim}{v}_k = -\frac{m}{e}\nu_e i\underset{\sim}{k} \times \delta\underset{\sim}{v}_k. \quad (10)$$

Substituting for $\omega_{\underset{\sim}{k}}$ from Eq. (5a) and $\nabla\cdot\delta\underset{\sim}{v}$ from Eq. (10) into Eq. (9) we obtain

$$-\frac{\psi}{1+\psi}\underset{\sim}{k}\cdot\underset{\sim}{v}_o\delta n_k \sim -i\frac{\delta v_z n_o}{L} + \frac{n_o\nu_e}{\Omega_e}\hat{x}\cdot\underset{\sim}{k} \times \delta\underset{\sim}{v}_k. \quad (11)$$

Thus, for $kL >> \Omega_e/\nu_e$ we obtain

$$\frac{|\delta v_k|}{v_o} \approx \frac{\Omega_e}{\nu_e}\frac{\psi}{1+\psi}\frac{\delta n_k}{n_o} = \frac{\nu_i/\Omega_i}{1+\psi}\frac{\delta n_k}{n_o} \equiv \beta\frac{\delta n_k}{n_o}. \quad (12)$$

Now

$$\left\langle\left|\frac{\delta n_k}{n_o}\right|^2\right\rangle = \int I(k)kdk \sim k^2 I_k \quad (13)$$

and

$$\Gamma_k \sim k\delta v \sim kv_o\beta\langle(\frac{\delta n}{n})^2\rangle^{1/2} \sim v_o\beta k^2 I_k^{1/2}, \tag{14}$$

which is precisely the result obtained by previous calculations using the modified DIA theory[4] and quoted in Eq. (7) except for a numerical factor of order unity. Thus, for our modes the "direct interaction approximation" is tantamount to accepting the basic idea behind Kolmogoroff's theory[7] that the eddy lifetime is just the time it takes for the eddy velocity to cover its length.

Furthermore, the spectrum law can also be obtained from the elementary Kolmogorov argument that the power transfer between the modes decays only as the collisional damping, i.e.,

$$k\frac{d}{dk}(\Gamma_k\, k^2 I_k) = \gamma_k\, k^2 I_k, \tag{15}$$

because the mode energy $E_k \approx (\delta n_k)^2 \sim k^2 I_k$. Since $\gamma_k \propto -k^2$ in the damped region from Eq. (5b) for $kL \gg 1$,

$$I_k \propto k^{-8/3}\,[k_o^{4/3} - k^{4/3}]^2, \tag{16}$$

and k_o is the cutoff wave number where I_{k_o} is heavily damped. For $k \ll k_o$, $I_k \propto k^{-8/3}$, since $(k/I_k)dI_k/dk = -(8/3)/[1-(k/k_o)^{4/3}]$ a power law is appropriate only for $k \ll k_o$.

We compare the results of our simple expression with actual numerical solutions[5] of the nonlinear version of Eqs. (1) - (4), for I_k in Fig. 1. We have adjusted our normalization constant to fit one point and chosen the value of k_o as the numerically determined cutoff value. It is seen that the agreement in shape of the data points and Eq. (16) is quite good.

In summary, we have taken the physical ideas of Kolmogoroff out of their context of three-dimensional fluid turbulence and have applied them to a novel two-dimensional situation of electrostatic plasma turbulence in the ionosphere where numerical solutions existed. We have found that the ideas carry over very well as is shown by their agreement in detail for the nonlinear self-damping of the modes as well as predicting spectra that also agree well. This indicates that the mixing length theory may be successfully applied to derive results for much more general situations where it may be difficult or impossible to carry out the DIA theory.

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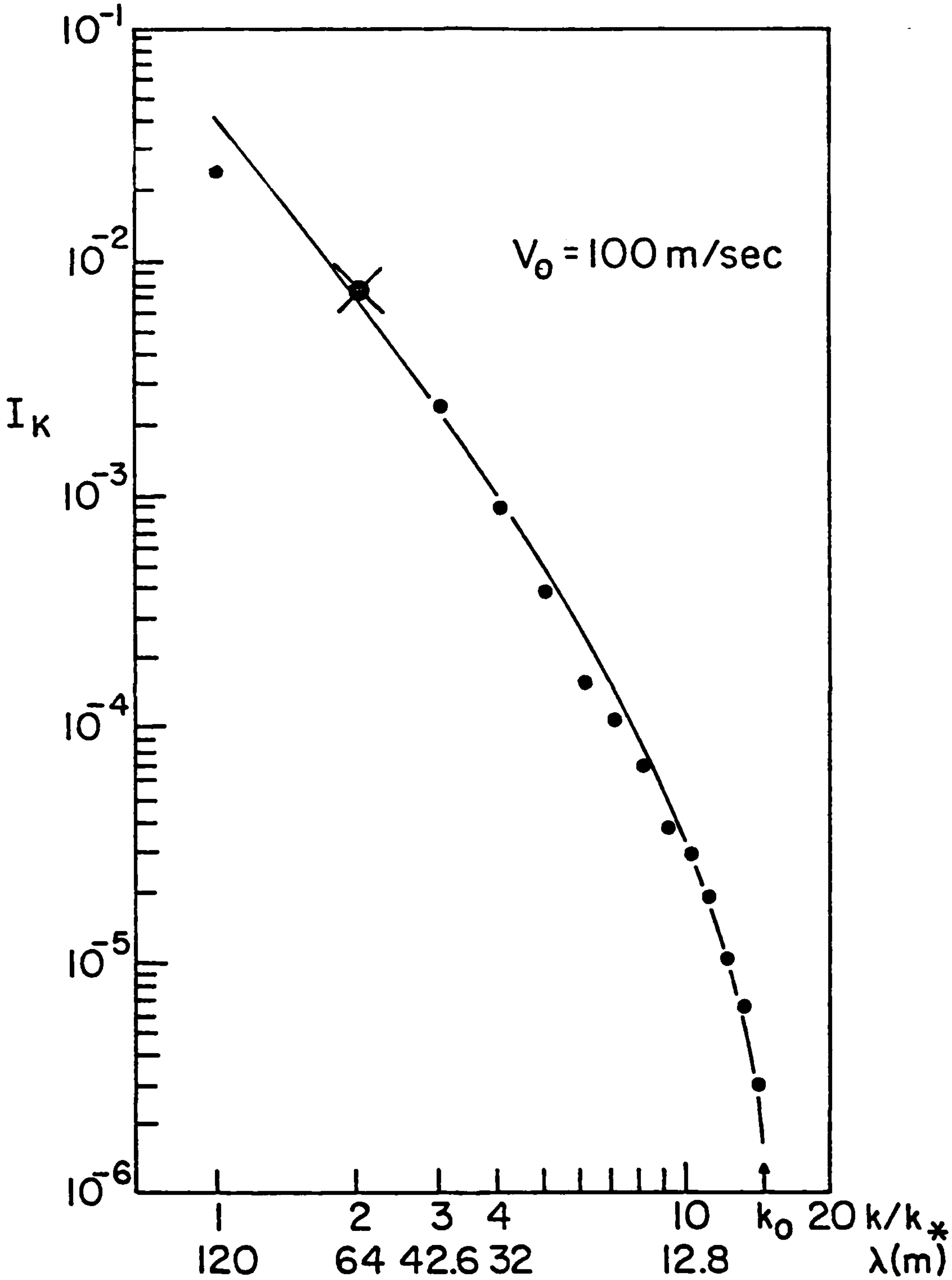
References

1. R. N. Sudan and M. J. Keskinen, Phys. Rev. Lett. 38, 966 (1977).
2. R. N. Sudan and M. J. Keskinen, Phys. Fluids 22, 2305 (1979).
3. R. H. Kraichnan, J. Fluid Mech. 5, 497 (1959).
4. See, for example, B. B. Kadomtsev, Plasma Turbulence (Academic Press, New York, 1965) Chapter III.
5. M. J. Keskinen, R. N. Sudan, and R. L. Ferch, J. Geophys. Res. 84, 1419 (1979).
6. B. B. Balsley and D. T. Farley, J. Geophys. Res. 76, 834 (1971); 78, 7471 (1973); D. T. Farley and B. B. Balsley, ibid., 78, 227 (1973).
7. S. Chandrasekhar, Ap. J. 110, 329 (1949).
8. A. Simon, Phys. Fluids 6, 382 (1963).
9. F. C. Hoh, Phys. Fluids 6, 1184 (1963).

Figure Captions

Fig. 1 Numerically obtained spectra I_k as a function k (Ref. 5). The continuous line is obtained from Eq. (16) after fitting at one point marked with a cross and the cutoff value $k_o = 15$.



10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
I_K
$V_0 = 100$ m/sec
1
2
3
4
10
k_0
20
k/k_*
120
64
42.6
32
12.8
λ(m)

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